Bayesian Games

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- They are analyzed with the notion of the Bayesian Nash equilibrium.

Example 1

Example 1

	C	D
C	2 1	0 0
D	0 0	1 2

Example 1

	C	D
C	2 1	0 0
D	0 0	1 2

	C	D
C	2 0	0 2
D	0 1	1 0

Example 1

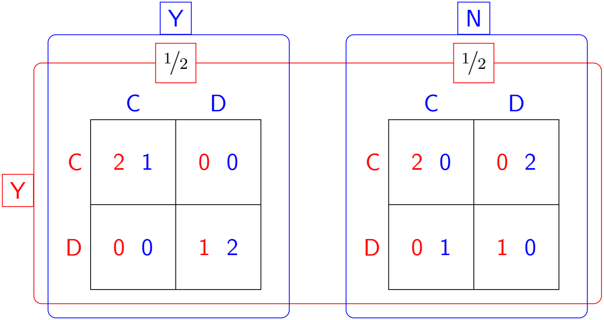
Y

	C	D
C	2 1	0 0
D	0 0	1 2

N

	C	D
C	2 0	0 2
D	0 1	1 0

Example 1



Example 2

	C	D
C	2 1	0 0
D	0 0	1 2

	C	D
C	2 0	0 2
D	0 1	1 0

	C	D
C	0 1	2 0
D	1 0	0 2

	C	D
C	0 0	2 2
D	1 1	0 0

Example 2

Y

2/3

	C	D
C	2 1	0 0
D	0 0	1 2

	C	D
C	2 0	0 2
D	0 1	1 0

1/3

	C	D
C	0 1	2 0
D	1 0	0 2

	C	D
C	0 0	2 2
D	1 1	0 0

Example 2

Y

2/3

	C	D
C	2 1	0 0
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1/3

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2/3

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1/3

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C	0 0	2 2
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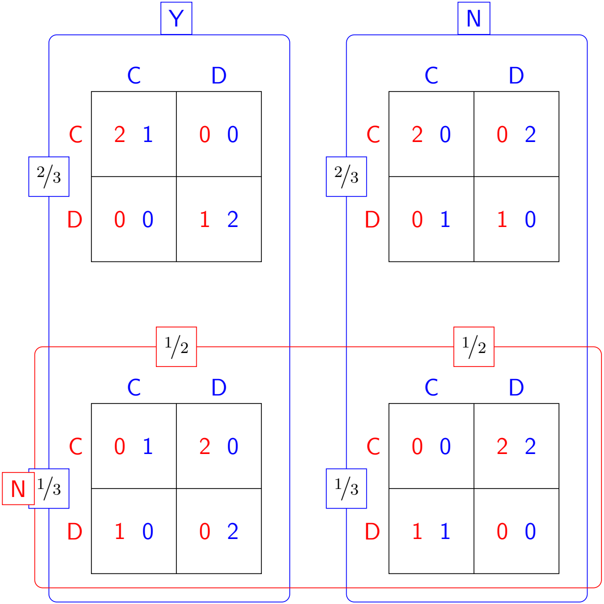
Example 2


Y
N
C D
C 2 1 0 0
D 0 0 1 2
2/3
C D
C 2 0 0 2
D 0 1 1 0
2/3
1/2
1/2
C D
C 0 1 2 0
D 1 0 0 2
N
1/3
C D
C 0 0 2 2
D 1 1 0 0
1/3

Example 2

Y | N

1/2 | 1/2

Y 2/3

	C	D
C	2 1	0 0
D	0 0	1 2

2/3

	C	D
C	2 0	0 2
D	0 1	1 0

1/2 | 1/2

N 1/3

	C	D
C	0 1	2 0
D	1 0	0 2

1/3

	C	D
C	0 0	2 2
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Bayesian Game

Definition

A **Bayesian game** consists of

- a set of **players**,
- a set of **states**,
- a set of **actions** for each player,
- a set of **signals** that a player may receive and a **signal function** that associates a signal with each state,
- for each signal that a player may receive, a **belief** about the states consistent with the signal (a probability distribution over the set of states with which the signal is associated), and
- a **Bernoulli payoff function** over pairs (a, ω), where a is an action profile and ω is a state, the expected value of which represents the player's preferences among lotteries over such pairs.

Bayesian Nash Equilibrium

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Bayesian Nash Equilibrium

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$$U_i\left(s_i, s_{-i}, t_i\right) = \sum_{\omega \in \Omega} P\left(\omega | t_i\right) u_i\left(\left(s_i\left(t_i\right), s_{-i}\left(\omega\right)\right), \omega\right).$$

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A **Nash equilibrium of a Bayesian game** is a Nash equilibrium of the strategic game defined as follows.

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- **Actions:** the set of actions for player (i, t_i) is the set of actions for player i in the Bayesian game.
- **Payoffs:** payoffs for player (i, t_i) as above.

Example 2 Revisited

Y: 1/2 (column player), Y: 2/3 (row player)

	C	D
C	2 1	0 0
D	0 0	1 2

N: 1/2 (column player), Y: 2/3 (row player)

	C	D
C	2 0	0 2
D	0 1	1 0

Y: 1/2 (column player), N: 1/3 (row player)

	C	D
C	0 1	2 0
D	1 0	0 2

N: 1/2 (column player), N: 1/3 (row player)

	C	D
C	0 0	2 2
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Unanimous Juries

- Blackstone's Formulation:

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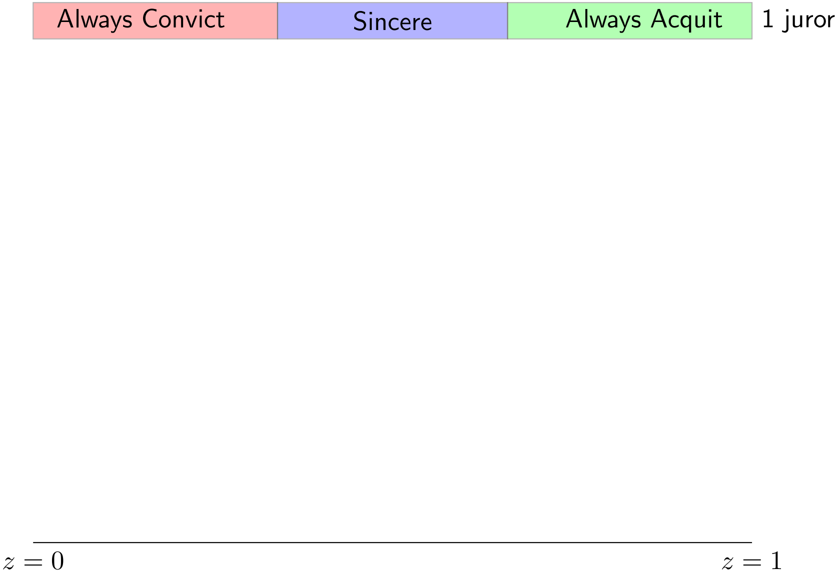
- Feddersen and Pesendorfer (1998): Do unanimous juries make it less likely that an innocent person is convicted?

Jury Voting

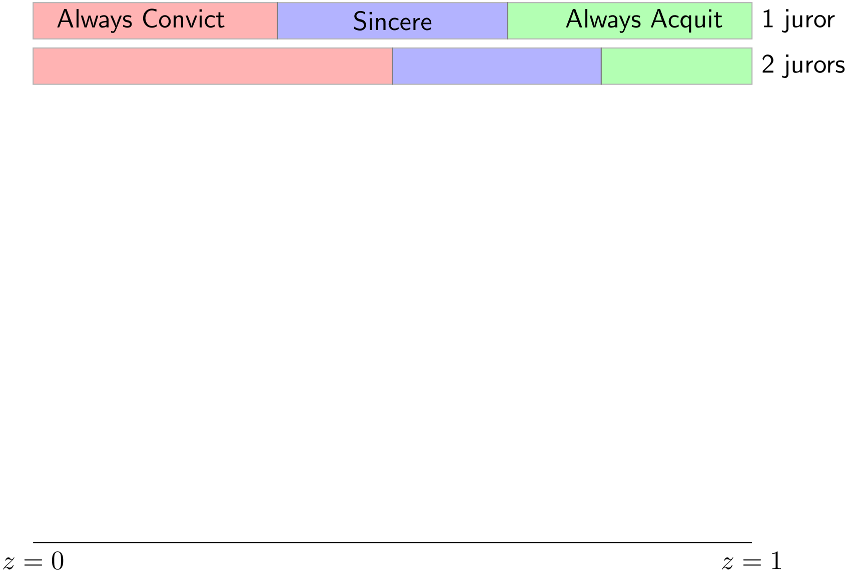
Always Convict | Sincere | Always Acquit

$z = 0$ $z = 1$

Jury Voting



Jury Voting



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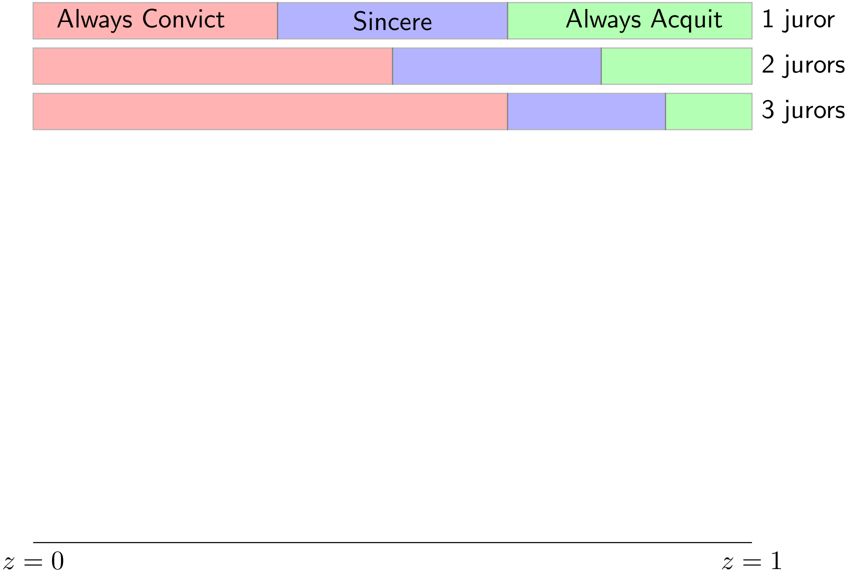
Always Convict
Sincere
Always Acquit
1 juror
2 jurors
3 jurors
$z = 0$
$z = 1$

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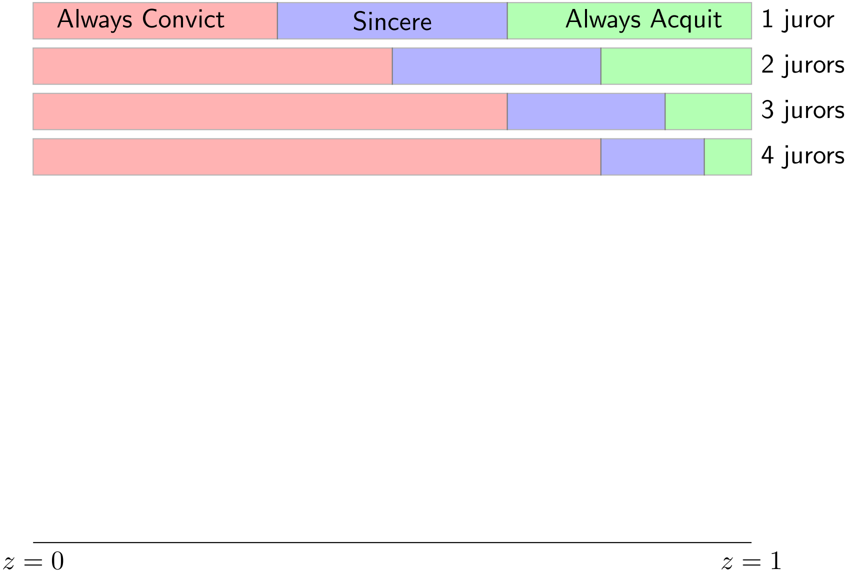
Always Convict
Sincere
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1 juror
2 jurors
3 jurors
4 jurors
$z = 0$
$z = 1$

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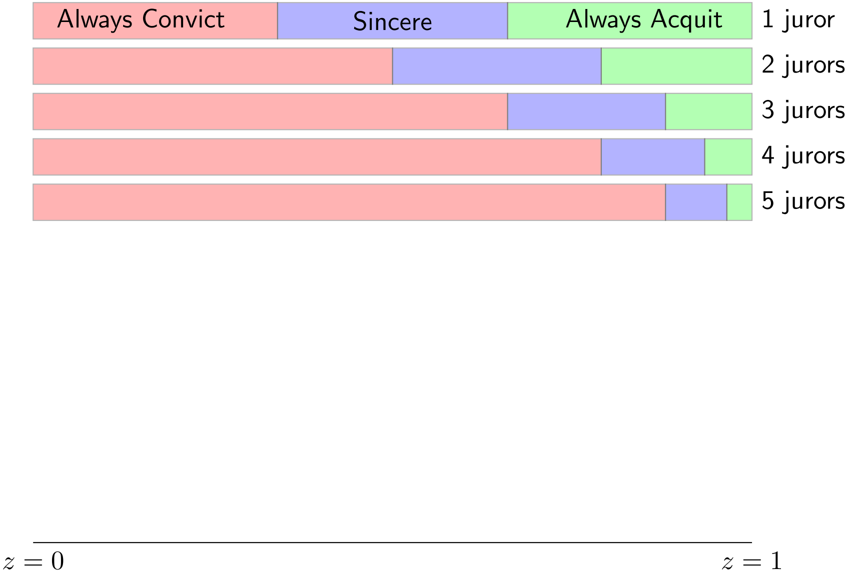
Always Convict
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1 juror
2 jurors
3 jurors
4 jurors
5 jurors
$z = 0$
$z = 1$

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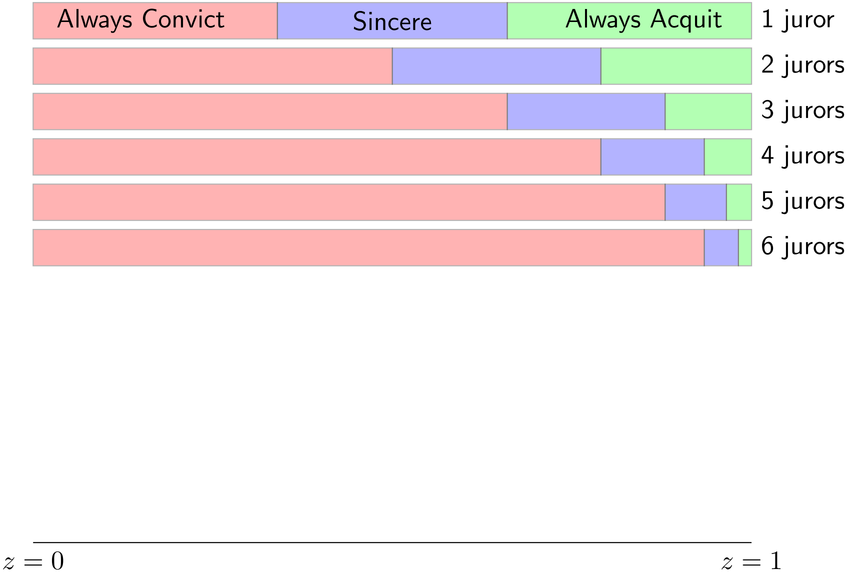
Always Convict
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1 juror
2 jurors
3 jurors
4 jurors
5 jurors
6 jurors
$z = 0$
$z = 1$

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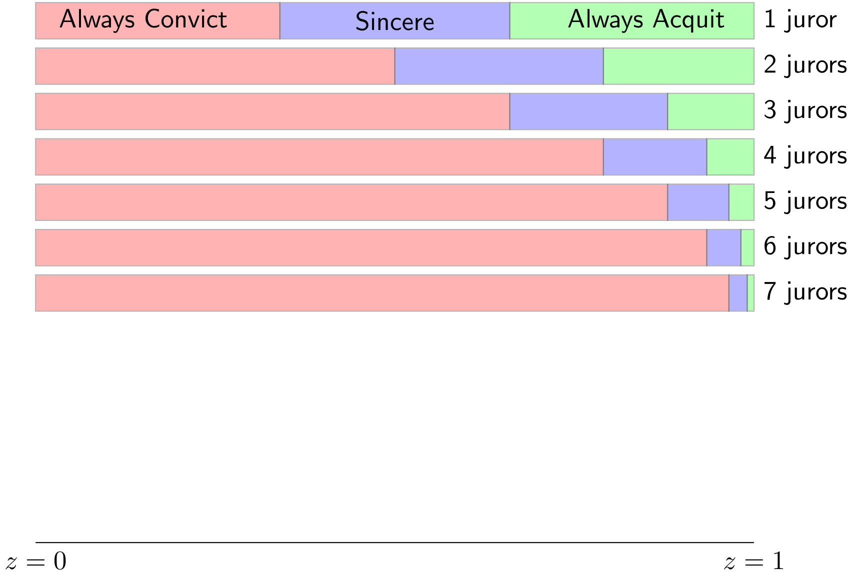

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4 jurors
5 jurors
6 jurors
7 jurors
$z = 0$
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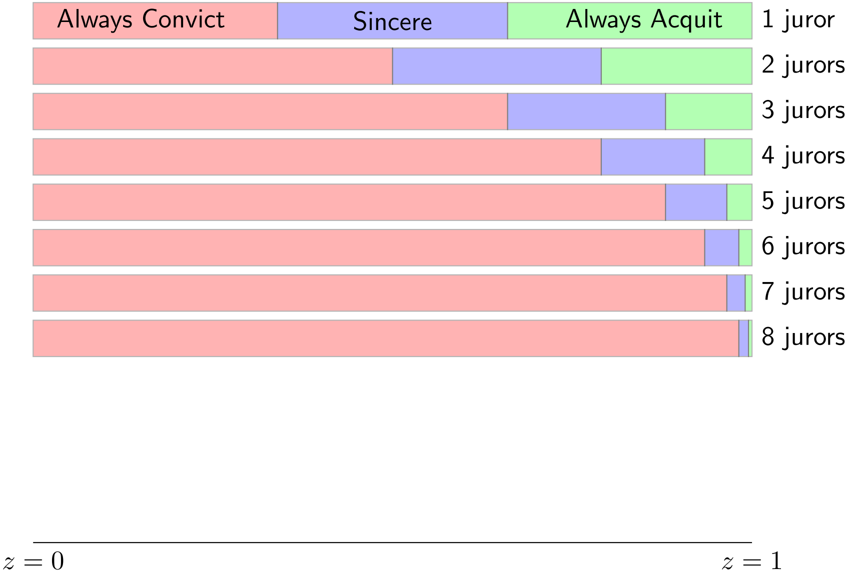
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8 jurors
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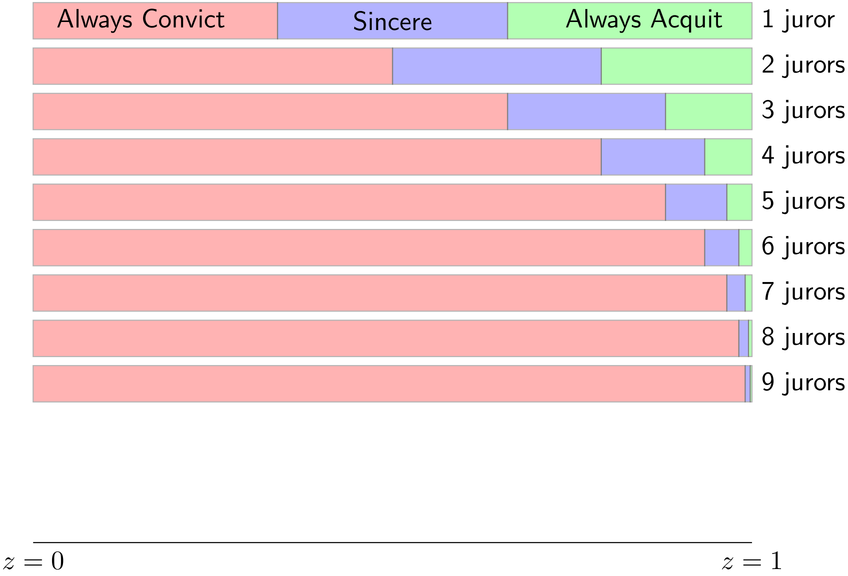
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9 jurors
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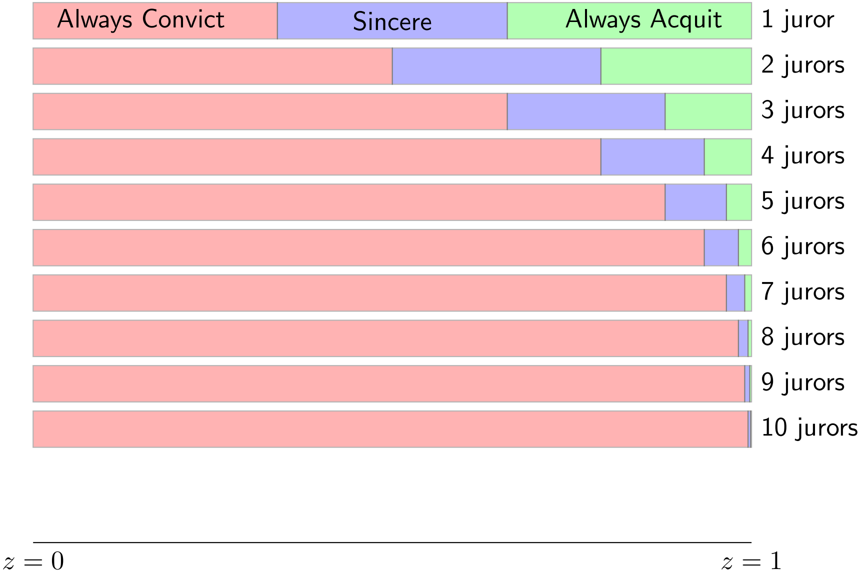
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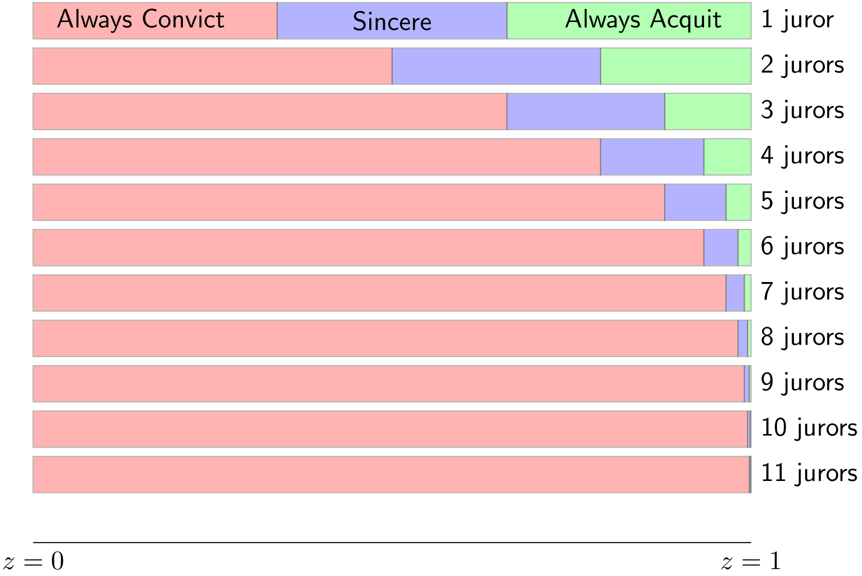

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11 jurors
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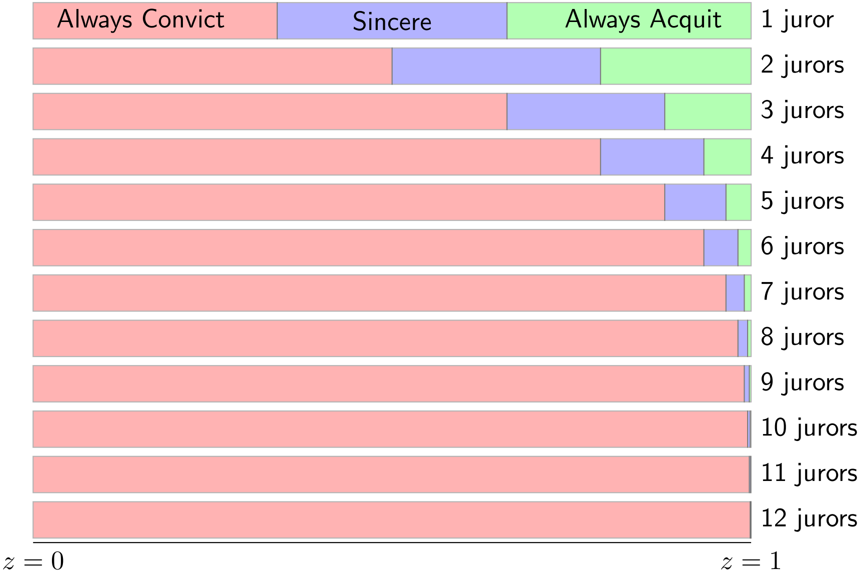
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1 juror
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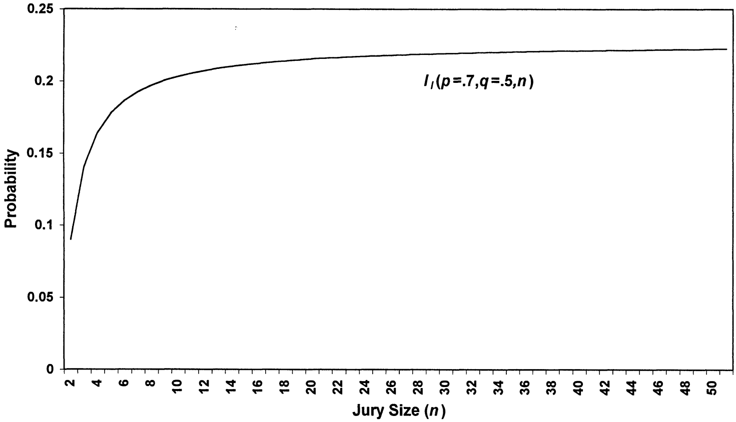
3 jurors
4 jurors
5 jurors
6 jurors
7 jurors
8 jurors
9 jurors
10 jurors
11 jurors
12 jurors
$z = 0$
$z = 1$

Strategic Voting

FIGURE 1. The Probability an Innocent Defendant Is Convicted as a Function of Jury Size



Strategic Voting

FIGURE 2. The Probability a Guilty Defendant Is Acquitted as a Function of Jury Size