Extensive Games with Perfect Information: Illustrations

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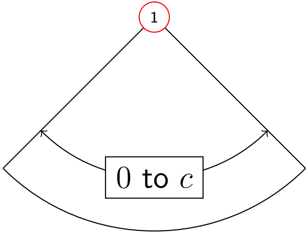

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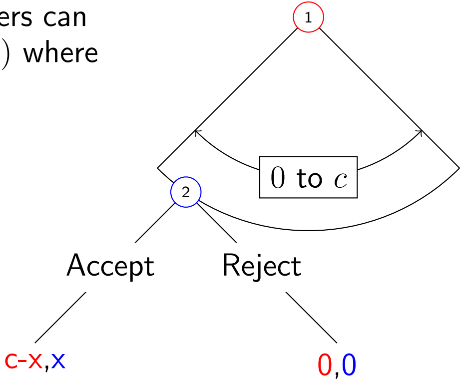

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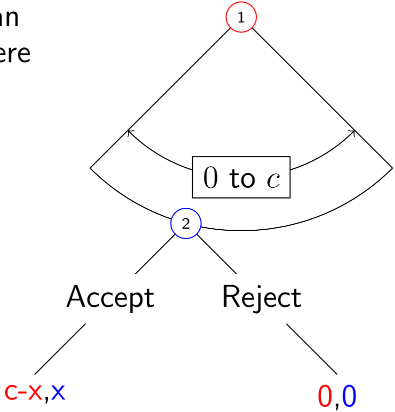

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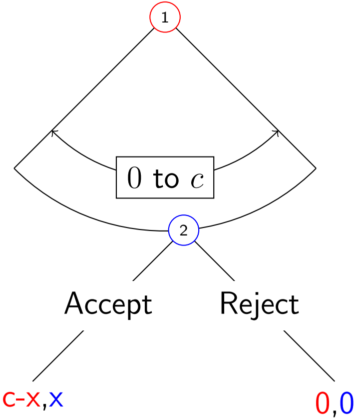

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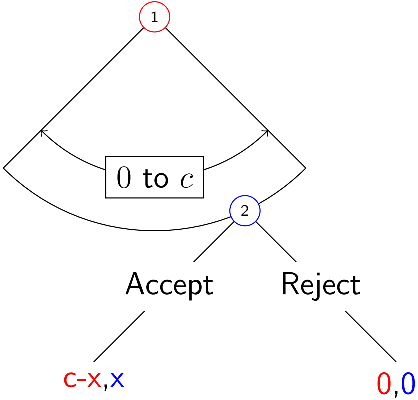
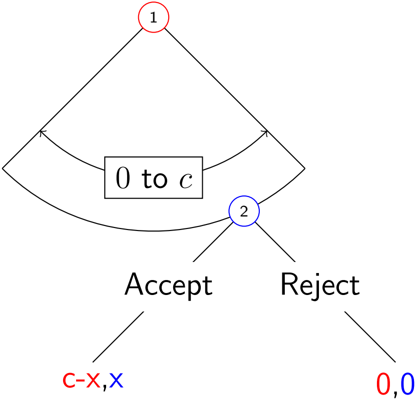


Ultimatum Game (Example)



Stackelberg Model

In the Stackelberg Model, firms set quantities sequentially.

Suppose Firm 1 is the leader and Firm 2 is the follower.

Firm 1 sets its quantity first, then, Firm 2 sets its quantity after observing what Firm 1's quantity was.

This model is different from Cournot, in which firms choose quantities simultaneously.

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  - Let $Q = q_1 + q_2$,
  - $P(Q)$ be the market price,
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  $\pi_i(q_i, q_{-i}) = q_i P(Q) - C_i(q_i)$ for $i = 1, 2$.

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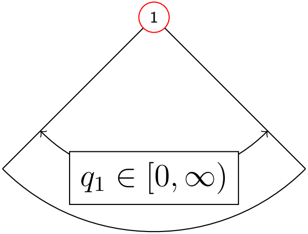

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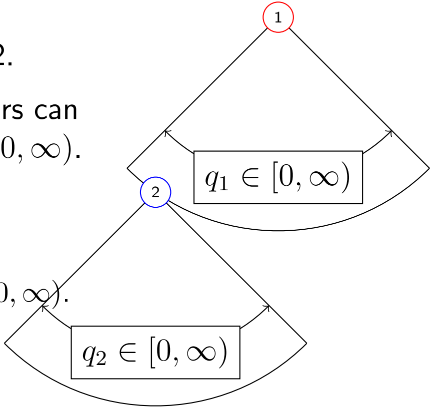

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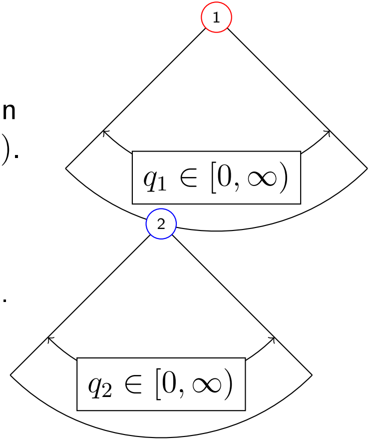

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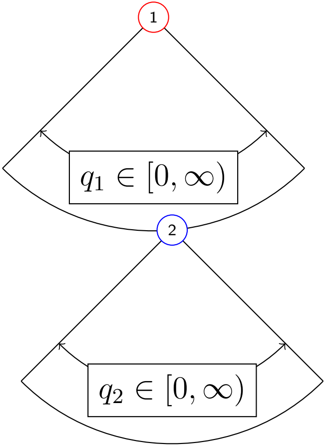

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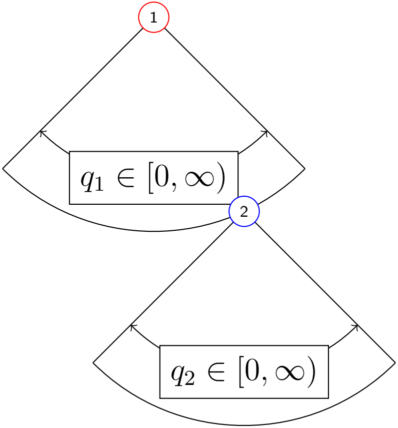


Stackelberg Best Response

Suppose Firm 1 (the leader) sets a quantity q_1.

Firm 2 (the follower)'s best response is $q_2^* = BR(q_1)$.

- This is the same as what we calculated in the Cournot model.
- Given that the inverse demand is $P = 2 - q_1 - q_2$ and $c = 1$, then, $q_2^* = 1/2 \cdot (1 - q_1)$ (check the refresher at the end).

  Firm 1 will anticipate Firm 2's best response; thus, it takes this best response into account when Firm 1 makes its quantity decision.

Stackelberg Solution

Firm 1 chooses q_1^* to maximize profit, where
$\pi_1 = TR - TC = (P(Q) - c)q_1$

$TR = (2 - q_1 - q_2^*)q_1 = (2 - q_1 - BR(q_1))q_1$

$= (2 - q_1 - 1/2(1 - q_1))q_1 = 3/2q_1 - 1/2q_1^2$

$MR = \frac{dTR}{dq_1} = 3/2 - q_1$

$MC = c = 1$

$MR = MC \Rightarrow 3/2 - q_1^* = 1 \Rightarrow q_1^* = 1/2$

$q_2^* = 1/2(1 - 1/2) = 1/4$

$P(Q) = 2 - q_1^* - q_2^* = 2 - 1/2 - 1/4 = 5/4$

The leader has an advantage over the follower.

Recall that in the Cournot solution both firms chose $1/3$.

Refresher of Cournot Model

For any given level of Firm 2's output level q_2^*, Firm 1 must choose an output level to maximize profits so that

$$\max_{q_1}[2-(q_1+q_2^*)]\cdot q_1 - q_1.$$

How does Firm 1 do it? As usual, by setting $MR = MC$.

$MR = \frac{dTR}{dq_1} = \frac{d(2q_1 - q_1^2 - q_2^* q_1)}{dq_1} = 2 - 2q_1 - q_2^*$

$MC = \frac{dC}{dq_1} = \frac{dq_1}{dq_1} = 1$

Cournot Model (Cont.)

So, we solve for Firm 1's profit maximizing q_1^* as a function of Firm 2's output level q_2^*. Thus,

$$MR = MC$$
$$2 - 2q_1^* - q_2^* = 1$$
$$q_1^* = \tfrac{1}{2}(1 - q_2^*)$$

This is called the **Cournot Best Response Function.**

Let us graph this ($q_2^* = 0 \rightarrow q_1^* = 1/2$ and $q_2^* = 1 \rightarrow q_1^* = 0$).

Cournot Model (Cont.)

Can you solve Firm 2's profit maximizing output q_2^* in terms of Firm 1's output level q_1^*?

Cournot Model (Cont.)

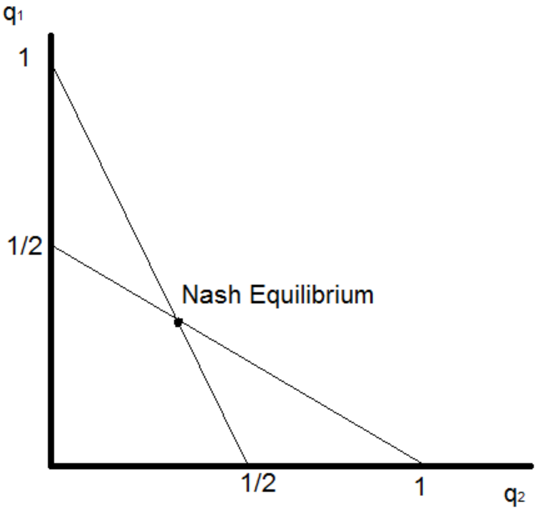
Can you solve Firm 2's profit maximizing output q_2^* in terms of Firm 1's output level q_1^*?

Firms choose a symmetric strategy so that
$q_2^* = \frac{1}{2}(1 - q_1^*)$.

Graph the best response function of Firm 2 as well.

To find the Nash Equilibrium, we want the two strategies to "match."

Cournot Solution Graph



Cournot Solution

Now, we solve a system of two equations and two unknowns.

By substitution,

$$q_1^* = \tfrac{1}{2}(1 - q_2^*)$$

$$\Rightarrow q_1^* = \tfrac{1}{2}\left(1 - \tfrac{1}{2}(1 - q_1^*)\right)$$

$$\Rightarrow q_1^* = \tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{4}q_1^*$$

$$\Rightarrow \tfrac{3}{4}q_1^* = \tfrac{1}{4} \Rightarrow q_1^* = \tfrac{1}{3}$$

Substituting it back, we find $q_2^* = (1 - 1/3)/2 = 1/3$.

So, both firms produce $1/3$.

Cournot Solution (Cont.)

So we have $q_1^* = q_2^* = \frac{1}{3}$.

Total quantity produced in the market: $Q = q_1^* + q_2^* = 2/3$.

Given the inverse demand function,

$P(Q) = 2 - Q = 2 - 2/3 = 4/3$.