On the Theory of Optimal Investment Decision
References

Objective

- The paper solves the problem of optimal investment decisions.

- Evaluates two competing rules of behavior proposed by economists to guide investment decisions: (a) the present-value rule, and (b) the internal-rate-of-return rule.

- Assumes a highly simplified situation in which the costs and returns are known with certainty.
Financial Manager

What is the job of a financial manager?

1. Decide what project the firm should undertake.
2. Decide how the firm should pay for it.
The 2-Period Investment-Consumption Model

- Hirshleifer aims to solve the problem of selecting the scale and the mix of investments to be undertaken.

- Assume there exists a given rate at which the financial manager may borrow that is unaffected by the amount of his borrowings.

- Assume there exists a given rate at which the financial manager can lend that is unaffected by the amount of the loans.

- Assume these two rates are equal.

- These are the conditions used by Fisher; they represent a perfect capital market.
• In Figure 1, the horizontal axis labeled $K_0$ represents the amount of actual or potential income (the amount consumed or available for consumption) in period 0.

• The vertical axis $K_1$ represents the amount of income in the same sense in period 1.

• The financial manager's decision problem is to choose, within the opportunities available to him, an optimal point on the graph; that is, an optimal time pattern of consumption.

• The financial manager’s starting point may conceivably be a point on either axis (initial income falling all in period 0 or all in period 1), such as points $T$ or $P$, or else it may be a point in the positive quadrant (initial income falling partly in period 0 and partly in period 1), such as points $W$ or $S'$. 
Figure 1

FIG. 1.—Fisher’s solution
• The financial manager is assumed to have a preference function relating income in periods 0 and 1.

• This preference function would be mapped in quite the ordinary way, and the curves $U_1$ and $U_2$ are ordinary utility-indifference curves from this map.

• There are production opportunities: there are real productive transfers between income in one time period and in another (what we usually think of as physical investment, like planting a seed).

• There are market opportunities: these are transfers through borrowing or lending (which naturally are on balance offsetting in the loan market).
Production & Market Opportunities

• Thus we may invest by building a house (a sacrifice of present for future income through a production opportunity) or by lending on the money market (a sacrifice of present for future income through a market or exchange opportunity).

• We could, equivalently, speak of purchase and sale of capital assets instead of lending or borrowing in describing the market opportunities.

• In Figure 1, a financial manager with a starting point at $Q$ faces a market opportunity illustrated by the dashed line $QQ'$. This is the market line.

• Finally, the curve $QSTV$ shows the range of productive opportunities available to a financial manager with starting point $Q$. This is the production possibility frontier.
Solution

- The financial manager’s objective is to climb onto as high an indifference curve as possible.

- Moving along the productive possibility frontier $QSTV$, the highest indifference curve is point $S$.

- But this is not the best point attainable, for the manager can move along $QSTV$ somewhat further to the point $R'$, which is on the market line $PP'$.

- The manager can now move in the reverse direction (borrowing) along $PP'$, and the point $R$ on the indifference curve $U_2$ is seen to be the best attainable.
Competing Rules of Optimal Investment Behavior

• Both are partially correct.

• The present-value rule, would have the financial manager adopt all projects whose present value is positive at the market rate of interest.

• The present-value rule tells us to invest until the highest such line is attained, which clearly takes place at the point $R'$. The rule says nothing about the financing (borrowing or lending) necessary to attain the final optimum at $R$.

• The internal-rate-of-return rule, would have the firm adopt any project whose required minimum annual rate of return is more than the discount rate.

• Evidently, this rule would have us move along $QSTV$ until it becomes tangent to a market line at $R'$. Again, nothing is said about the borrowing or lending then necessary to attain the optimum.
What is the Moral of this Model?

Though simple, the model emphasizes several interesting points.

• The existence of financial markets leads to a choice of optimal solutions of investment and consumption choices with higher levels of utilities for the agents as compared with the case where no such markets exist.

• The movement to point $R'$ in Figure 1 does not depend on preferences: a prodigal manager (i.e. a manager who typically prefers to borrow money in order to spend today more than what (s)he has) and a miser manager (someone who prefers to save today in order to have more tomorrow) will agree on the optimality of the move. Once at $R'$, and hence at the new level of wealth $PP'$, they will obviously choose differently.
Fisher’s Separation Theorem (1930)

Fisher’s Separation Theorem

The optimal production decision is determined by an objective market criterion independently of individuals’ subjective preferences that define their consumption decisions.

In other words, the capital investment criterion has nothing to do with the individual preferences for current versus future consumption.
Implications

Any investment policy can thus be seen as a sequential decision in two phases. In the first phase, one chooses the level of investment. In the second, a decision is made about the intertemporal consumption path. Therefore, the level of investments is decided independently of the consumption preferences, but the level of possible consumption depends on the investment decision.

Another important implication of this result is that there is no conflict of interest between shareholders and managers: shareholders can delegate decisions to professional managers who, as long as they maximize net present value, will act in the best interest of shareholders.
More General Questions

- Is it true that there is no conflict of interest between the owner of an enterprise (namely the shareholders) and whoever runs it (typically the managers)?

- Do managers have the correct incentive to maximize shareholders’ wealth?

- Questions as such have been analyzed in a branch of modern literature referred to as Principal-Agent Problems.

- The Principal-Agent Problems are concerned with the difficulties in motivating one party (the agent) to act on behalf of another (the principal). Common examples of this relationship include corporate management (agent) and shareholders (principal), or politicians (agent) and voters (principal).
Imperfect Capital Markets

• While describing the previous model, we have always assumed that capital markets are perfect. This is the case when the borrowing and the lending interest rates are equal. This is typically not the case in reality.

• The borrowing and lending rates are still assumed to be constant independently of the amounts taken or supplied.

• However, it is now assumed that these rates are not equal; that is, the borrowing rate is higher than the lending rate.
• In Figure 2, there is the same preference map, of which only the isoquant $U_1$ is shown.

• There are now, however, two sets of market lines over the graph; the steeper (dashed) line represents borrowing opportunities, and the flatter (solid) line represents lending opportunities.

• The heavy solid lines show two possible sets of production possibility frontiers, both of which lead to solutions along $U_1$. 
**Figure 2**

![Graph](image)

**Fig. 2.** Extension of Fisher’s solution for differing borrowing and lending rates.
Starting with amount $OW$ of $K_0$, a financial manager with a production opportunity $WVW'$ would move along $WVW'$ to $V$, at which point he would lend to get to his time-preference optimum; that is, the tangency with $U_1$ at $V'$. 
The curve $STS'$ represents a more productive possibility; starting with only $OS$ of $K_0$, the financial manager would move along $STS'$ to $T$ and then borrow backwards along the dashed line to get to $T'$; that is, the tangency point with $U_1$. 
Another Possibility

- A manager on the $K_0$ axis will never stop moving along the production possibility frontier in the direction of greater $K_1$ as long as the marginal rate of transformation is above the borrowing rate. Nor will he ever push along the locus beyond the point where the marginal rate of transformation falls below the lending rate.

- But what if borrowing or lending decrease utility! This can only mean that a tangency of the production possibility frontier and an indifference curve took place when the marginal rate of transformation was somewhere between the lending and the borrowing rates.

- In this case neither lending nor borrowing is called for; that is, the optimum is reached directly by equating the marginal rate of transformation with the marginal rate of substitution.
• These solutions are illustrated by the division of Figure 3 into three zones.

• In Zone I, the borrowing rate is relevant. Tangency solutions with the market line at the borrowing rate like that at $T$ are carried back by borrowing to tangency with a utility isoquant at a point $T'$.

• In Zone III, the lending rate is relevant. The productive solution involves tangency with a lending market line (like $V$), which is then carried forward by lending to a final tangency optimum with a utility isoquant at a point $V'$.

• In Zone II, the solutions occur when a production possibility frontier like $QRQ'$ is steeper than the lending rate throughout Zone III but flatter than the borrowing rate throughout Zone I. Therefore, such a locus must be tangent to one of the indifference curves somewhere in Zone II like point $R$. 
**Figure 3**

**FIG. 3.**—Three solution zones for differing borrowing and lending rates.