Structural Models of Decision Rules

References

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- Stahl, D. O., and P. W. Wilson: On Players' Models of Other Players: Theory and Experimental Evidence, *Games and Economic Behavior*, 10 (1995), 218-254.
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Motivation

- Empirically, on many occasions, individuals stay away from the Nash equilibrium prediction.
- This does not say that the notion of Nash equilibrium is wrong.
- However, it does assume a high degree of rationality on the part of the individual, which may not be realistic.
- Structural models try to deal with these issues to provide accurate predictions.

Quantal Response Equilibrium

- McKelvey and Palfrey (1995) introduced the Quantral Response Equilibrium (QRE).
- In this model, beliefs are consistent with the observed distribution, but there is a perturbation in the individual's best response; thus, individuals choose better responses.
- Thus, strategies with higher expected payoffs are chosen quite often but not always.
- QRE assumes a specific functional form. Typically, we choose a logit specification.

Quantal Response Equilibrium (Cont.)

- In a logit equilibrium, player's strategies are chosen according to the probability distribution

$$P_{i,j} = \frac{e^{\lambda EU_{i,j}(P_{-i})}}{\sum_k e^{\lambda EU_{i,k}(P_{-i})}}.$$

- $P_{i,j}$ is the probability of player i choosing strategy j. $EU_{i,j}(P_{-i})$ is the expected utility to player i of choosing strategy j under the belief that other players are playing according to the probability distribution P_{-i}.

- The non-negative parameter λ can be thought of as the rationality parameter. As $\lambda \to 0$, players become completely non-rational, and play each strategy with equal probability. As $\lambda \to \infty$, players become perfectly rational, and play approaches a Nash equilibrium.

Quantal Response Equilibrium (Cont.)

- We use maximum-likelihood estimation to choose the rationality parameter λ.
- For example, if subjects play right with probability $\frac{1}{2}$, then MLE finds the λ such that the QRE predicts play of right with probability as close to $\frac{1}{2}$ as possible.

Example (Logit Specification)

	Left	Right
Up	3, 3	0, 0
Down	0, 0	1, 1

- The Row player believes that the Column player will play left with probability q.
  - $EU_{Row,Up} = 3 \cdot q + 0 \cdot (1 - q) = 3q.$
  - $EU_{Row,Down} = 0 \cdot q + 1 \cdot (1 - q) = 1 - q.$
- Row player's strategies are chosen according to the probability distribution

$$P_{Row,Up} = \frac{e^{\lambda EU_{Row,Up}}}{e^{\lambda EU_{Row,Up}} + e^{\lambda EU_{Row,Down}}}.$$

- Compute the probability for the Column player.

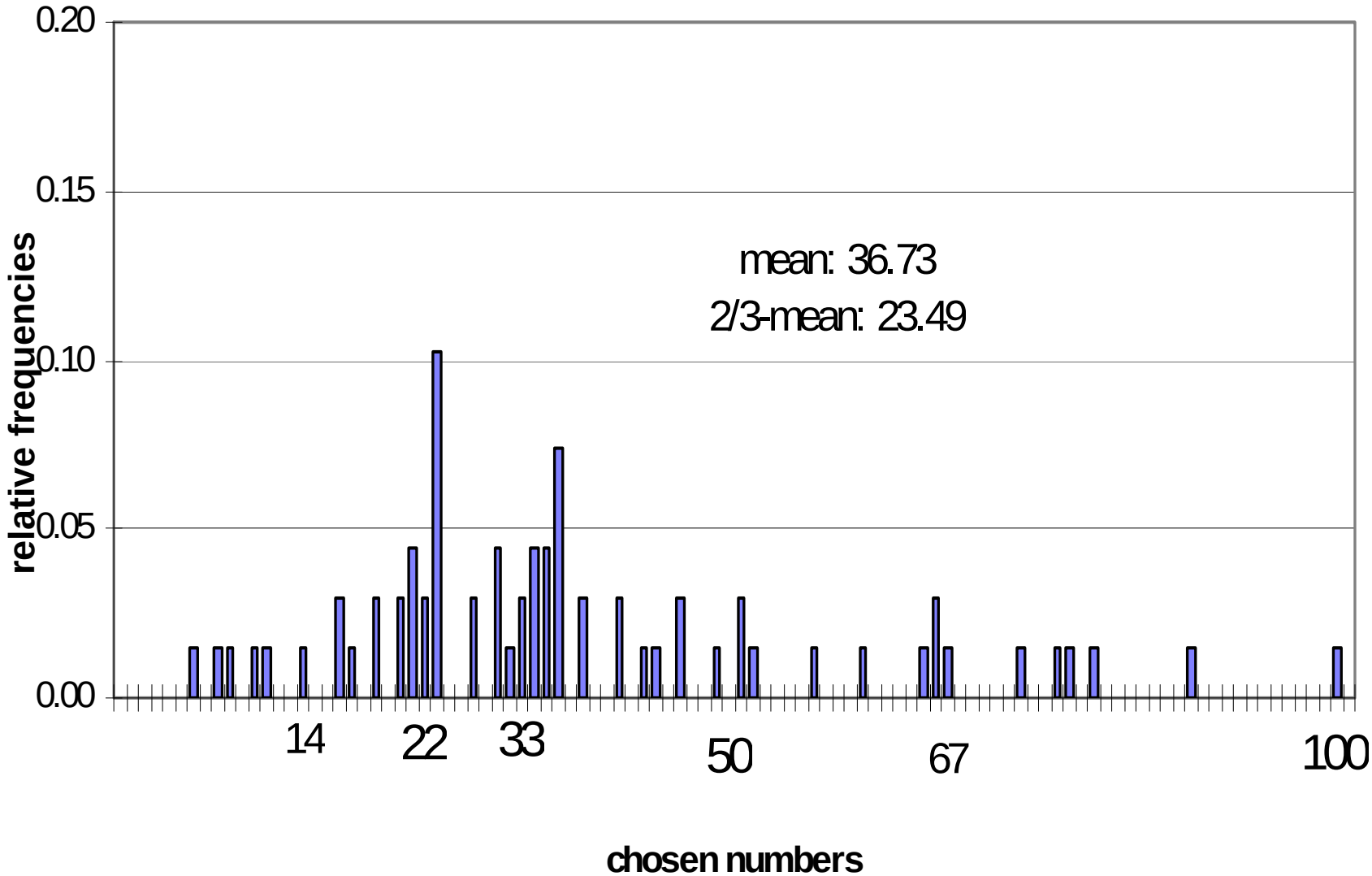
Level-k

- Stahl and Wilson (1995) introduced the Level-k model in which players have different degrees of strategic sophistication.
- The model assumes a distribution of types where each type corresponds to a level of reasoning.
- Thus, each level of reasoning best responds to the level below it.
- Consider next the p-Beauty Contest game.

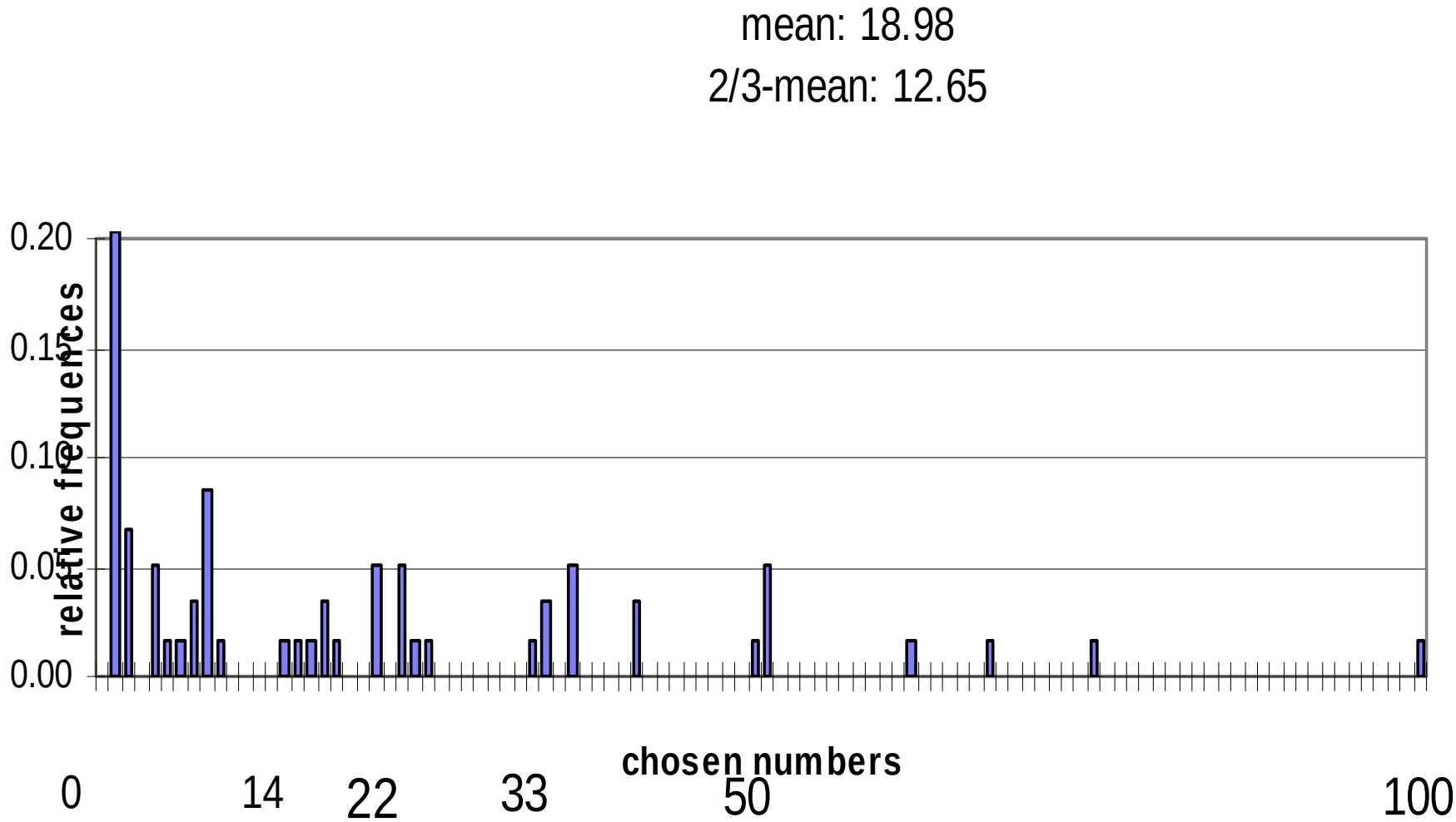
The p-Beauty Contest Game ($p = \frac{2}{3}$)

Choose a number between 0 and 100. The winner is the person whose number is closest to $\frac{2}{3}$ times the average of all chosen numbers. The winner gets a fixed prize of \$20. In case of a tie the prize is split amongst those who tie.

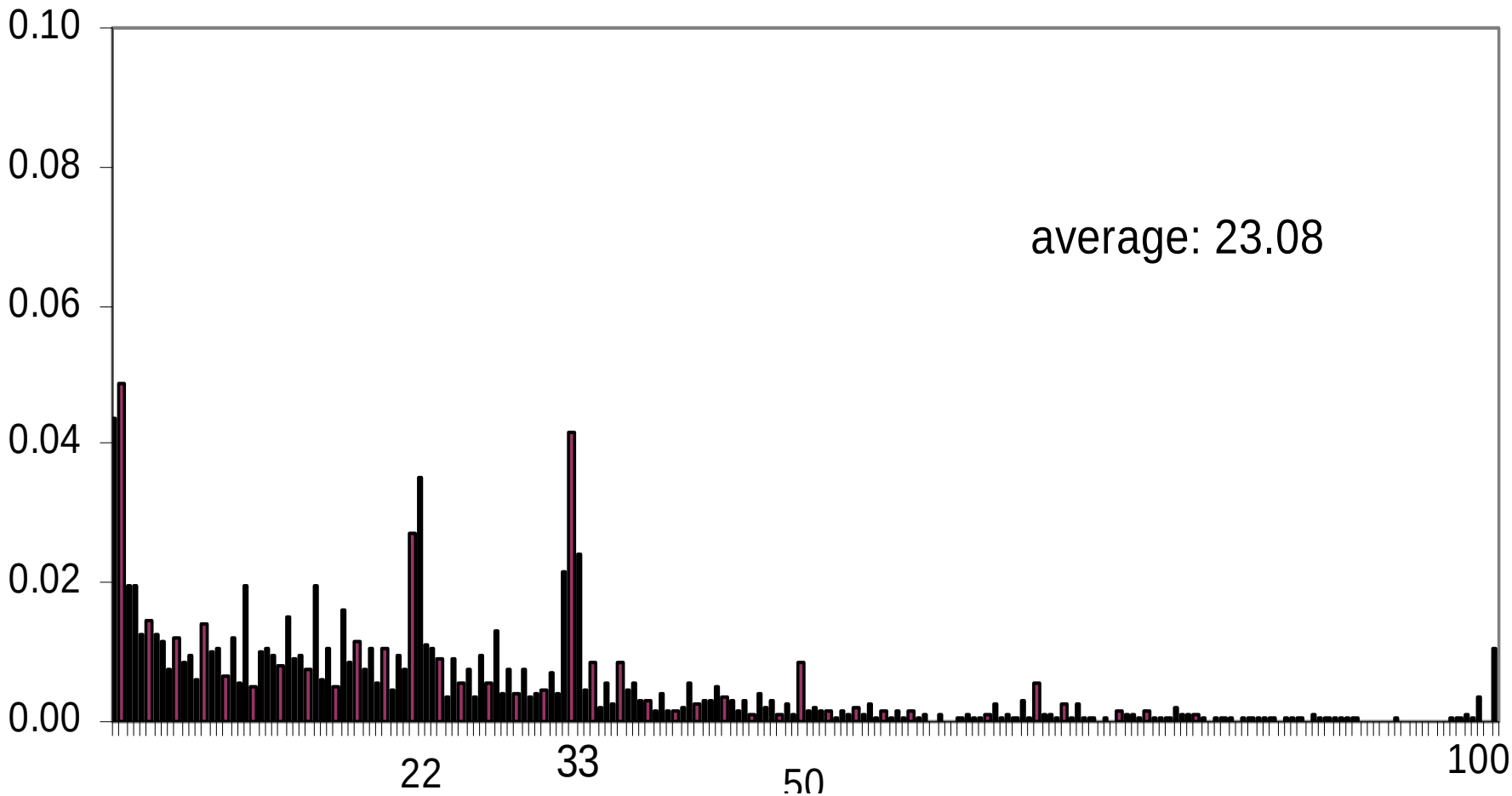
Laboratory Results



Theorists' Results



Newspaper Readers' Results



The p-Beauty Contest Game

"... professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

Keynes (1936, p. 156)

Depth of Reasoning

- Consider the following sequence of reasoning for the $\frac{2}{3}$-Beauty Contest game.
  1. If I think the other players will play 50, then I should play $33\frac{1}{3}$.
  2. If I think the other players think that everyone will play 50 and so will play $33\frac{1}{3}$, then I should best respond to this and choose $22\frac{2}{9}$.
  3. If I think that the other players will initially think that everyone will play 50, and will consider playing $33\frac{1}{3}$ but they will think that others have done the same reasoning and therefore play $22\frac{2}{9}$, then I should best respond by playing $14\frac{22}{27}$.
  4. ...
- Continuing this reasoning, then I should play 0. This is the Nash equilibrium.

Level-k Thinking

- There is a distribution of types in the population.
  - You have a type equal to:
  - Level 0 $\rightarrow$ Non-strategic (plays at random).
  - Level 1 $\rightarrow$ best responds to Level 0.
  - Level 2 $\rightarrow$ best responds to Level 1.
  - Level 3 $\rightarrow$ best responds to Level 2.
  - ...
  - Level k $\rightarrow$ best responds to Level k-1.
- Maximum-likelihood parameter estimates allow us to place each individual in his respective type given his choice.

Issues with Level-k

- The model has lots of degrees of freedom.
  - What is Level 0?
  - What is the distribution of types?
- The model has low predictive power.
- Are types fixed?
  - Georganas, Healy and Weber (2015) examine whether the Level-k model of strategic behavior generates reliable cross-game predictions at the individual level.
  - The authors estimate the type in each game but find no correlation in the estimated type.
- The model does not respond to incentives and learning.

Cognitive Hierarchy

- Camerer, Ho and Chong (1995) introduced the Cognitive Hierarchy (CH) model.
- In this model, Level-k best responds, not to Level k-1 alone, but to a mixture of lower-level types, where the type frequencies are treated as a parametrized Poisson distribution where the mean is τ.
- Although this specification seems more natural than the simpler Level-k specification, which specification better describes people's behavior is an empirical question.
- Notice that in this model, type beliefs may not be consistent with the observed distribution, however there are no perturbations in the best responses; thus, the noise is in the distribution in contrast to the QRE.