Disclaimer: For this homework we will assume that all payoffs are representing ordinal preferences, so we will only consider pure-strategy Nash equilibria.

1. Consider the following game:

	L	R
U	α,α	1,5
D	5,1	β,β

Determine all values of α and β for which the game is a:

a) Prisoner's Dilemma

b) Battle of the Sexes

c) Stag Hunt Game

d) Chicken Game

e) Matching Pennies Game

2. Consider the following game: Two players, player 1 has actions U or D player 2 has actions L and R. If players choose (U, L), then, they both receive \$4. If they choose (D, R), then, they receive \$0. If they choose (D, L), then, player 1 gets \$6 and player 2 gets \$1. If they choose (U, R), then, player 1 gets \$1 and player 2 gets \$6.

a) Suppose each player only cares about their own payoff. What game is this? Determine the Nash equilibria.

b) Now suppose that each player cares about their own payoff but also their opponents payoff (they are altruistic) in the following manner: $u_i(a_i, a_{-i}) = m_i(a_i, a_{-i}) + \alpha m_{-i}(a_i, a_{-i})$. That is player i's benefit from a given outcome is his payoff plus α times his opponent's payoff. For what values of α is the game the same as the game in part (a). For what values of α the game is not the same as in part (a). Find the Nash equilibrium.

3. Suppose you are going out to dinner with a group of people. There are two items on the menu: a steak (which costs \$50) and a salad (which costs \$5). Everyone values a salad at \$6 (because they are delicious) and values a steak at \$10. The group decides that they are going to split the bill evenly amongst everyone eating dinner. An individual's payoff is, therefore, the value that they get for an item minus the price they have to pay.

a) Suppose you are the only one eating dinner, what will you order.

b) Suppose that there are two people eating dinner, what is the Nash equilibrium? What is your payoff in the Nash equilibrium?

c) Suppose that there are N people eating dinner, what is the Nash equilibrium? (**Hint:** Given that everyone else keeps ordering what they are ordering, what is your best response?)

4. You are working on an assembly line with N workers (including yourself). The assembly line produces a product that is either high quality or low quality. For every high quality product, each worker on the the assembly line receives \$10. For every low quality product, each worker on the assembly line receives \$5. Each worker can decide whether to put in high effort (which costs them \$3) or low effort (which costs them nothing). The assembly line produces a high quality product if at least k individuals exert high effort.

a) Suppose you are the only person in the assembly line (and $k = 1$), what will you do?

b) Suppose there are $N = 2$ people in the assembly line and $k = 2$, write out the game in the normal form, tell me what game this is, and determine the Nash equilibria.

c) Suppose there are $N = 2$ people in the assembly line and $k = 1$, write out the game in the normal form, tell me what game this is, and determine the Nash equilibria.

d) Suppose there are $N = 10$ people in the assembly line and $k = 10$. Determine the Nash equilibria.

e) Suppose there are $N = 10$ people in the assembly line and $k = 9$. Determine the Nash equilibria.

5. There are d democrats and r republicans voting in the US Presidential election. Each person decides whether to vote or not vote. The voters get a payoff of 2,1,0 if their preferred candidate wins, ties, or loses (respectively). Voters that vote incur a cost of $0 < c < 1$ (which leads to respective payoffs of $2 - c$,$1 - c$,$-c$).

a) Suppose that $d = r = 1$. Write out the normal form game table, tell me what game this is, and determine the Nash equilibrium.

b) Now suppose $d = r > 1$, find the set of Nash equilibria. **Hints:** Is there any NE in which everyone votes. Is there any NE in which there is a tie and not everyone votes? Is there any NE in which one of the candidates wins by one vote? Is there any NE in which one of the candidates wins by 2 or more votes?

c) Finally, suppose $d < r$, find the set of Nash equilibria.

6. Your professor lost two homework assignments, one which was yours and one which belonged to one of your classmates. Your professor remembers that the grade was

the same on both assignments, but doesn't remember what it was. The students know the true grade because they have checked the answer key, and determined which problems they answered correctly and which problems they missed. The professor devises the following plan. Each student can tell the professor a grade between 0 and 100 (integers only). If both students write down the same number, then, the professor will take that as the true value, and each student will be assigned that grade. If the students write down different numbers, then the professor takes the lower number to be the true grade (call it TG) and, then, gives a grade of $TG - 2$ to the student that reported the high grade (because they may have been lying to increase their grade) and a grade of $TG + 2$ to the student that reported the lower grade (because they were being more truthful).

a) Assume that players can only choose either 5 or 10 for their grades. Write down the game in the normal form, determine which game this is, and find the Nash equilibrium.

b) Assume that players can only choose either 9 or 10 for their grades. Write down the game in the normal form, determine which game this is, and find the Nash equilibrium.

c) Now assume that players can choose any integer between (and including) 0 and 100. What is the Nash equilibrium?