1. Consider the Cournot duopoly exercise we did in class. Let q_M be the optimal quantity for a monopolist (that is, if $q_2 = 0$, then firm 1 wants to produce q_M). Suppose that each firm can only set two quantities: the Cournot equilibrium quantity or half of q_M. Assume that $\alpha > c$ and $\alpha - c > 1$.

   a) Write out the normal form game table of this two player game.

   b) Determine the Nash equilibrium(a) of this game.

   c) What game is this?

2. Consider an industry with two firms in which each firm chooses to produce $q_i \in [0, \infty)$, with a total production of $Q = q_1 + q_2$. The demand in the industry is given by $P(Q) = \alpha - 2Q$ if $Q < \alpha/2$ and $P(Q) = 0$ if $Q \geq \alpha/2$. Each firm faces the same cost function, $C(q_i) = cq_i^2$ where $c > 0$ is a constant.

   a) Write out the profit function for firm i, $\pi_i(q_i, q_{-i})$.

   b) If firm 2 produces $q_2 = 0$, how much does firm 1 want to produce?

   c) What is the best response of firm 1, when firm 2 produces $q_2 \in [0, \infty)$.

   d) What is the Cournot equilibrium.

   e) What is the Cournot equilibrium if there are $n > 2$ firms?

3. Pepsi and Coke are *horizontally differentiated products*, which means that they are close substitutes, but not exact substitutes. Therefore, the price of Pepsi affects the demand of Coke and vice versa, but some people would still prefer to buy Coke at a higher price than Pepsi. Suppose Coke and Pepsi choose prices $p_i \in [0, \infty)$. The demand for Coke given the prices of Coke and Pepsi is

$$D_C(p_C, p_P) = 100 - 5p_C + 8p_P.$$

   The demand for Pepsi given the price of Coke and Pepsi is,

$$D_P(p_C, p_P) = 80 - 5p_P + 10p_C.$$

   Each firm faces a linear cost function $C(q_i) = cq_i$. (**Note:** The demand functions are linear in both prices which may not be the most realistic assumption because it means that if Pepsi sets a price of ∞, then, there will be an infinite demand for Coke. This assumption is only made to simplify the problem.)

   a) Write out the profit function for each firm: $\pi_C(p_P, p_C)$ and $\pi_P(p_P, p_C)$.

   b) If Coke sets a price of 0, what price does Pepsi want to set?

   c) What is Pepsi's best response to Coke's price $p_C \in [0, \infty)$?

   d) What is Coke's best response to Pepsi's price $p_P \in [0, \infty)$? (**Reminder:** The firms *are not* symmetric in this problem.)

   e) What is the Nash equilibrium?

4. Consider an auction with n bidders, ordered in a such a way that their values are ordered from highest to lowest ($v_1 > v_2 > \cdots > v_n$). Assume the same tiebreaker that we used in class (in tie, the bidder with higher value wins).

   a) Show that in a first price auction, the following must be true in a Nash equilibrium:

      - The two highest bids are the same.
      - One of the bids is submitted by player 1.
      - The highest bid is between v_2 and v_1 (including the endpoints).

   b) If there are only two bidders, find all Nash equilibria of a second-price auction.

   c) Find the set of all Nash equilibria in an all-pay auction with n players.