1. Consider a variant of the ultimatum game, with indivisible units. Two people use the following procedure to allocate two identical objects. One person proposes an allocation (both objects go to person 1, both go to person 2, or one goes to each). The other person either accepts or rejects. In the event of a rejection, neither person receives either object. Each person cares only about the number of objects she obtains.

   a) How many terminal and non-terminal histories are there?

   b) How many strategies does each player have?

   c) Model this situation as an extensive form game with perfect information and find the Subgame Perfect Nash equilibria.

   d) Model this situation as a simultaneous move game and find the Nash equilibria.

   e) Are there any payoffs that occur in a Nash equilibrium but don't occur in a subgame perfect Nash equilibrium.

2. Consider an industry with two firms; firm 1 and firm 2. The firms set quantities $q_i \in [0, \infty)$. The total production in the market is $Q = q_1 + q_2$. The market price is determined by the demand curve, $P(Q) = \alpha - Q$ if $Q < \alpha$ and $P(Q) = 0$ otherwise. Each firm faces a cost function of $C_i(q_i) = q_i^2$.

   a) Suppose that firm 1 chooses $q_1 \in [0, \infty)$ first, firm 2 sees q_1, and then sets $q_2 \in [0, \infty)$. (Stackelberg) Find the subgame perfect equilibria.

   b) Can you find a Nash equilibrium in which firm 1 produces $q_1 = 0$. (**Remember: a strategy is a complete contingent plan.**)

   c) Can you find a Nash equilibrium in which firm 1 produces their profit maximizing quantity (monopoly quantity).

   d) Determine all quantities that firm 1 could produce in a Nash equilibrium.

3. In the ultimatum game that we talked about in class, player 1 offers some amount x between (and including) 0 and c ($x \in [0, c]$), and then player 2 decides to accept or reject the offer. Consider the following variant of the ultimatum game. In this variant, players don't just care about their payoff, they also care about inequality (how different their payoff is from their opponent's payoff). In particular, when players receive x_1 and x_2 respectively, player i gets a utility of,

$$u_i(x_1, x_2) = x_i - \beta_i |x_1 - x_2|$$

   Assume that $\beta_i > 0$ and $c = 1$.

   a) Given β_2, what offers will player 2 accept?

   b) For each value of $\beta_1 > 0$, what payoffs x_1 and $x_2 = c - x_1$ give player 1 the highest utility?

   c) Find the set of subgame perfect equilibria.

   d) Now, assume that $\beta_1 = 0$ and that player 2 has $\beta_2 = 0$ with probability p and $\beta_2 = 1$ with probability $(1 - p)$. Model this as an extensive form game with chance moves in which player 1 makes and offer first, then chance chooses the type of player 2, and finally player 2 decides whether to accept or reject. Determine the subgame perfect Nash equilibria of this game.