1. Army 1, of country 1, must decide whether to attack army 2, of country 2, which is occupying an island between the two countries. In the event of an attack, army 2 may fight, or retreat over a bridge to its mainland. Each army prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies.

   a) Model this situation as a simultaneous move game and find the Nash equilibria.

   b) Model this situation as an extensive form game with perfect information and find the Subgame Perfect Nash equilibria.

   c) Consider the slight modification of the game, where before army 1 decides what to do, army 2 can burn the bridge (which means that they must stay and fight). Model this situation as an extensive form game with perfect information and find the Subgame Perfect Nash equilibria.

2. Consider a variant of the illustration we studied in class (entry into a monopolized industry). There are two firms. Firm 1 is currently operating in the industry, and firm 2 is deciding whether to enter the industry or not. In this case, rather than firms setting quantities, firms set prices (like in a Bertrand Duopoly model). The firm with the lowest price gets all of the demand, and if the prices are equal, then the firms split the demand. Assume that $Q(p) = \alpha - p$ when $p < \alpha$ and $Q(p) = 0$ otherwise. Also assume that each firm has a cost of $C_i(q_i) = cq_i$.

3. Determine how many strategies each player has and find all pure and mixed SPNE of the following game:

   $P(\emptyset) = P(RL) = P(LL) = 1$ $\quad$ $P(R) = P(L) = 2$
   $u(LLL) = (7, 8)$ $\quad$ $u(LLR) = (7, 2)$
   $u(LR) = (1, 5)$ $\quad$ $u(RLL) = (3, 3)$
   $u(RLR) = (3, 7)$ $\quad$ $u(RR) = (5, 4)$

4. If the prisoner's dilemma is played once, each player has just 2 pure strategies. In a repeated prisoner's dilemma, players get to see the action of their opponent from the previous period before making their choice.

   (a) Suppose the repeated prisoner's dilemma is played for 2 rounds. How many pure strategies does each player have?

   (b) Suppose the repeated prisoner's dilemma is played for T rounds (where T is an arbitrary positive integer). How many pure strategies does each player have? Prove your answer.

   (c) What is the subgame-perfect Nash equilibrium?

   (d) If possible, find another pure-strategy Nash equilibrium that is not subgame-perfect; if not possible, explain.